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The ΔI = 1 Energy Staggering in Odd Superdeformed Nuclei ^{191}Hg, ^{193}Hg and ^{193}Pb

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ABSTRACT: The energy formula expressed in terms of rotational, vibrational and perturbation terms is built for three pairs of signature partners in odd superdeformed (SD) nuclei ^{191}Hg, ^{193}Hg and ^{193}Pb. A fitting procedure is employed to parameterize spins of the signature partner pairs in terms of their observed γ-ray energies. With the suggested level spins for each of the studied SD bands, the model parameters are determined. The best fitted parameters have been used to calculate E2 transition γ-ray energies, dynamic and kinematic moments of inertia and the calculated results agreed with the experimental data very well. It indicates that our proposed energy formula is powerful in describing the signature parterres. There is a significant increase in nuclear moments of inertia with increasing rotational frequency. We study theoretically the ΔI = 1staggering phenomenon by means of proposed staggering function depending on the dipole and quadruple γ-ray transition linking the two signature partner bands. A large amplitude staggering pattern is found in the three signature partner pairs.

I. INTRODUCTION

Since the experimental discovery of the first discrete superdeformed rotational band (SDRB) at high spins in the nucleus ^{152}Dy in 1986[1], a large number superdeformed (SD) bands have now been found in nuclei in several mass regions A~30, 60, 80, 130, 150, and 190[2, 3]. Recently the study of superdeformation is one of the most exciting areas in nuclear physics. Because of the large quadruple deformation, the decay sequences in SD bands are usually found to be completely dominated by stretched E2 transitions. However, level spins in most of SD bands were not determined experimentally. Because of the regular behavior of their transition energies, their spins were predicted theoretically by various approaches [4-13]. For many SD bands in mass A~ 190 region, the dynamical moments of inertia show a typical smooth rise with increasing rotational frequency. This behavior was attributed to the gradual alignment of quasi particle pairs occupying specific high N intruder orbital (namely $j_{15/2}$ neutrons and $i_{13/2}$ protons) in the presence of weak pair correlations.

Another interesting features connected to the SD nuclei in mass A~190 region is the measurement of the magnetic properties. The branching ratios of M1 transition between signature partner SD bands and the cross-talk between them were measured in ^{193}Hg and ^{193}Pb[14,15]. It was seen that majority of SD bands observed in odd-A nuclei in the mass A~190 region are signature partners. They exhibit a ΔI = 1energy staggering or signature splitting [16-25]. In previous papers [26-30], we investigated the ΔI = 1 staggering in signature partner pairs of odd-A SD bands by extract the differences between the average transition $E_\gamma(I+2\rightarrow I)$ and $E_\gamma(I\rightarrow I-2)$ energies in one band and the transition $E_\gamma(I+1\rightarrow I-1)$ energies in the signature partner. The purpose of this work is to investigate the ΔI = 1staggerring in signature partner pairs of odd-A nuclei by using a staggering function depends on the dipole and quadruple transition energies. In section 2 the outline of the model to be used is described in terms of rotational, vibrational and perturbation terms. Section 3 is devoted to study the concept of ΔI = 1energy staggering in SD bands. Numerical calculations and discussions for describing the structure of SD bands in ^{191}Hg, ^{193}Hg and ^{193}Pb nuclei are presented in section4.

II. THEORETICAL FRAMEWORK

In framework of phenomenological rotational-vibrational model (RVM), the energies E within a band of an axial symmetric nucleus exhibit a smooth dependence on angular momentum I can be expressed in a simple expression in the form

$$E(I) = AI(I+1) + BI + CI^3 + E_0$$

The first term is the rotational energy with the inertial parameter $A = \hbar^2/2J$ where J is the kinematic moment of inertia about an axis perpendicular to the nuclear symmetry axis and is related to the angular momentum I. The second term is the vibrational energy with B is the vibrational constant. The third term is a perturbation term proportional to the cubic power of angular momentum. The last term is a bandhead energy.

Experimentally, the γ-ray transition energies $E_{\gamma 2}$ (I→I-2) are commonly observed, thus $E_{\gamma 2}$ within a band can be given by the parabolic function

$$E_{\gamma 2}(I) = E(I) - E(I-2) = C_2 I^2 + C_1 I + C_0$$

where $C_2 = 6C$

$C_1 = 4A - 12C$

$C_0 = -2A + 2B + 8C$

Therefore, a quadratic fit through the variation of E_γ with spin I is obtained. The spin values start from $I = I_0 + 2$ where I_0 is the bandhead spin. The fit then evaluates the model parameters C_2, C_1 and C_0.

In order to investigate the ΔI = 1energy staggering, we need the dipole transition energy, that is the transition energy $E_{\gamma 1}$ from spin I to spin (I-1) which is given by

$$E_{\gamma 1}(I) = E(I) - E(I-1) = b_2 I^2 + b_1 I + b_0$$

where $b_2 = 3C$

$b_1 = 2A - 3C$

$b_0 = B + C = 0$

One can determine the initial values of the three parameters C_2,C_1 and C_0 from the three first transitions $E_\gamma(I_0)$, $E_\gamma(I_0+2)$ and $E_\gamma(I_0+4)$ and considering these as a trial values for the fetting. It is easily to verity that

$$C_2 = \frac{1}{8}[E_\gamma(I_0+4) - 2E_\gamma(I_0+2) + E_\gamma(I_0)]$$

$$C_1 = \frac{1}{4}[-(I_0+1)E_\gamma(I_0+4) + (2I_0+4)E_\gamma(I_0+2) - (I_0+3)E_\gamma(I_0)]$$

$$C_0 = \frac{1}{8}\{I_0(I_0+2)E_\gamma(I_0+4) - 2I_0(I_0+4)E_\gamma(I_0+2) + [8 + I_0(I_0+6)]E_\gamma(I_0)\}$$

The rotational frequency ħω, the dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia will be calculated from the γ-ray transition energies E_γ(I) with the following definitions

$$\hbar\omega(I) = \frac{E_\gamma(I+2\to I) + E_\gamma(I\to I-2)}{4}$$

$$J^{(2)}(I) = \frac{4}{E_\gamma(I+2\to I) - E_\gamma(I\to I-2)} \qquad \hbar^2 MeV^{-1}$$

$$J^{(1)}(I) = \frac{2I-1}{E_\gamma(I\to I-2)} \qquad \hbar^2 MeV^{-1}$$

with E_γ in unit of MeV.

It is seen that, while the extracted $J^{(1)}$ depends on the spin I proposition, ħω and $J^{(2)}$ does not.

III. ANALYSIS OF ΔI = 1 STAGGERING IN SD BANDS

It was found that many SD bands observed in odd-A nuclei in the A~ 190 region are signature partners and exhibit ΔI = 1signature splitting with large amplitude [16-30]. To investigate this ΔI = 1 staggering, we will propose the following staggering function

$$Y(I) = \frac{2I-1}{I}\,\frac{E(I) - E(I-1)}{E(1) - E(I-2)} - 1 = \frac{2I-1}{I}\,\frac{E_{\gamma 1}(I)}{E_{\gamma 2}(I)} - 1$$

The plot of Y(I) versus I is the most simple way to identify the crossing of the two signature partners. Although one expects linear behavior Y(I) = 0 for pure rotator according to the usual I(I + 1) formula, the band exhibit a zigzag behavior or perturbation with large amplitude staggering. In previous papers [26-30], we investigated the ΔI = 1 staggering in signature partner pairs of odd SD bands, by extracting the difference between the averaged transitions I +2→I and I→I-2 energies in one band and the transition I+1→I-1 energies in the signature partner

$$\Delta^2 E_\gamma(I) = \frac{1}{2}[E_\gamma(I+2\to I) + E_\gamma(I\to I-2) - 2E_\gamma(I+1\to I-1)]$$

IV. NUMERICAL RESULTS

Our selected data set includes three signature partner pairs in Hg and Pb namely ^{191}Hg (SD2, SD3), ^{193}Hg (SD3, SD4), and ^{193} Pb (SD3, SD4). The experimental transition energies are taken from Ref. [3].

To parameterize the level spins for each SD bands, we assumed various values for the bandhead spin I_0 and the model parameters are adjusted by using a simulated search program in order to obtain a minimum root mean square deviation χ defined by

$$\chi = \frac{1}{N}\Big[\sum_{I_i} \Big| \frac{E_\gamma^{exp}(I_i) - E_\gamma^{cal}(I_i)}{\Delta E_\gamma^{exp}(I_i)} \Big|^2\Big]^{1/2}$$

of the calculated energies $E_\gamma^{cal}$from the observed ones E_γ^{exp} , where N is the number of the data points entering into the fetting procedure and $\Delta E_\gamma^{exp}(I_i)$ is the experimental errors in γ-ray transition energies. The fitting procedure was repeated with bandhead spin I_0 fixed at the nearest half integer. The calculated bandhead spin I_0 and the resulting best model parameters C_2, C_1, C_0, b_2 and $b_1$and the lowest observed E_γ are listed in Table 1.

Table 1: Bandhead spin proposition I_0 and the best model parameters adopted from the best fit method for our selected SD bands.

S.D Bands	I_0 (ħ)	$E_\gamma(I_0+2\rightarrow I_0)$ (KeV)	C_2(KeV)	C_1(KeV)	C_0(KeV)	b_2(KeV)	b_1(KeV)
^{191}Hg(SD2)	10.5	252.4	-0.055066	21.762846	-10.881423	-0.027533	10.85387
^{191}Hg(SD3)	11.5	272.0	-0.062234	21.820192	-10.910096	-0.031117	10.878979
^{193}Hg(SD3)	9.5	233.5	-0.063950	21.978037	-10.989018	-0.031975	10.957043
^{193}Hg(SD4)	10.5	254.0	-0.062911	21.954720	-10.977360	-0.031455	10.945904
^{193}Pb(SD3)	12.5	291.5	-0.046108	21.583440	-10.791720	-0.023054	10.768665
^{193}Pb(SD4)	13.5	313.4	-0.064998	21.938188	-10.969094	-0.032499	10.936594

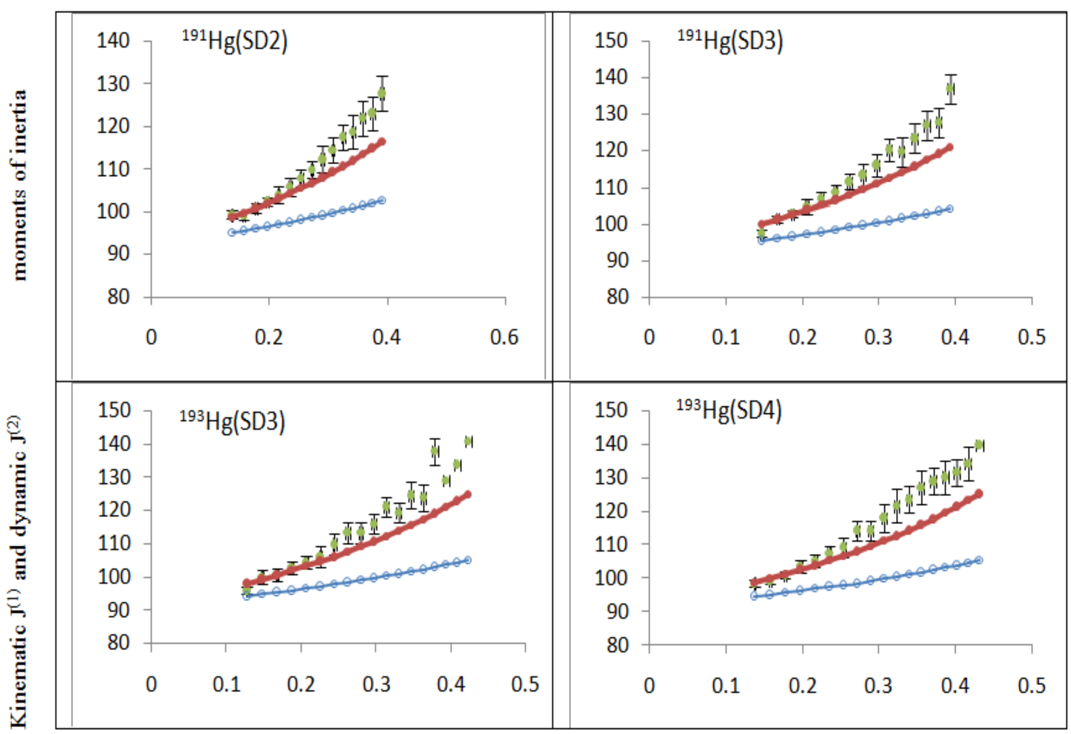
The bandhead spin assignments are consistent with previous works [27, 29, 31]. Using the adopted I_0 and the optimized model parameters, The transition energies E_γ (I), the rotational frequency ħω, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are calculated. Table 2 compare the calculated E_γ with experimental data [3]. Very good agreement has been obtained. Figure (1) presents the dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia for our three pairs of signature partner as a function of rotation frequency ħω, compared with the experimental data. For all cases $J^{(2)}$ moments of inertia exhibit a smooth increase with frequency. The average values of $J^{(2)}$ for bands 3 and 4 of ^{193}Pb (105.066 and 111.241 $\hbar^2 MeV^{-1}$ respectively) are smaller than the corresponding values for bands 2 and 3 of ^{191}Hg (110.986, 114.547 $\hbar^2 MeV^{-1}$ respectively) and for bands 3 and 4 of ^{193}Hg (116.286, 117.115 $\hbar^2 MeV^{-1}$ respectively).

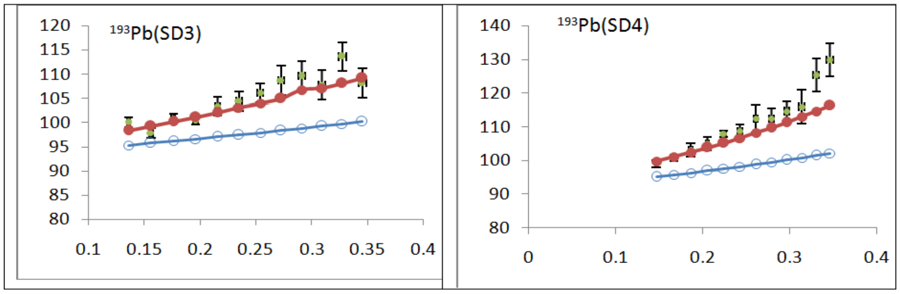
Table 2: The calculated γ-ray transition energies E_γ (I) for our selected SD signature partners and comparison with experimental data [3]. The bandhead spin and the model parameters are listed in Table (1).

^{191}Hg(SD2)				**^{191}Hg(SD3)**			
I (ħ)	**$E^{cal}(I)$ (KeV)**	**$E_\gamma^{cal}(I)$ KeV**	**$E_\gamma^{exp}(I)$ (KeV)**	**I (ħ)**	**$E^{cal}(I)$ (KeV)**	**$E_\gamma^{cal}(I)$ KeV**	**$E_\gamma^{exp}(I)$ (KeV)**
10.5	634.9345			11.5	754.0190		
12.5	887.4845	252.5500	252.5500	13.5	1026.3393	272.7203	272.0
14.5	1180.5867	293.1022	292.7	15.5	1338.6904	312.3511	313.1
16.5	1513.8005	333.2138	333.1	17.5	1690.5745	351.8841	352.55
18.5	1886.6853	372.8848	372.75	19.5	2081.4936	390.9191	391.5
20.5	2298.8007	412.1154	411.8	21.5	2510.9499	429.4563	429.7
22.5	2749.7061	450.9054	450.3	23.5	2978.4455	467.4956	467.1
24.5	3238.9610	489.2549	488.1	25.5	3483.4826	505.0371	503.9
26.5	3766.1248	527.1638	525.2	27.5	4025.5633	542.0807	539.7
28.5	4330.7571	564.6323	561.6	29.5	4604.1897	578.6264	575.0
30.5	4932.4173	601.6602	597.2	31.5	5218.8639	614.6742	609.5
32.5	5570.6649	638.2476	632.15	33.5	5869.0881	650.2242	642.7
34.5	6245.0593	674.3944	666.2	35.5	6554.3644	685.2763	676.1
36.5	6955.9600	7100.1007	699.9	37.5	7274.1949	719.8305	708.5
38.5	7700.5265	745.3665	732.7	39.5	8028.0817	753.8868	740.0
40.5	8480.7183	780.1918	765.2	41.5	8815.5270	787.4453	771.3
42.5	9295.2948	814.5765	796.5	43.5	9636.0329	820.5059	800.5
^{193}Hg(SD3)				^{193}Hg(SD4)			
9.5	535.8507			10.5	646.9318		
11.5	769.1517	233.3010	233.5	12.5	900.5585	253.6267	254.0
13.5	1043.2112	274.0595	275.2	14.5	1194.6975	249.1390	294.6
15.5	1357.5177	314.3065	315.2	16.5	1528.8455	334.1480	334.9
17.5	1711.5596	354.0419	354.9	18.5	1902.4991	373.6536	374.5
19.5	2104.8253	393.2657	393.8	20.5	2315.1551	412.6560	413.1
21.5	2536.8031	431.9778	432.1	22.5	2766.3102	451.1551	451.1
23.5	3006.9815	470.1784	469.8	24.5	3255.4611	489.1509	488.3
25.5	3514.8489	507.8674	506.2	26.5	3782.1045	526.6434	524.9
27.5	4059.8937	545.0448	541.5	28.5	4345.7372	563.6327	559.9

Continued…

^{191}**Hg(SD2)**				^{191}**Hg(SD3)**			
I (ħ)	$E^{cal}(I)$ **(KeV)**	$E_{\gamma}^{cal}(I)$ **KeV**	$E_{\gamma}^{exp}(I)$ **(KeV)**	**I (ħ)**	$E^{cal}(I)$ **(KeV)**	$E_{\gamma}^{cal}(I)$ **KeV**	$E_{\gamma}^{exp}(I)$ **(KeV)**
29.5	4641.6042	581.7105	576.8	30.5	4945.8558	600.1186	594.9
31.5	5259.4689	617.8647	611.3	32.5	5581.9570	636.1012	628.8
33.5	5912.9762	653.5073	644.3	34.5	6253.5376	671.5806	661.7
35.5	6601.6145	688.6383	677.8	36.5	6960.0443	706.5567	694.1
37.5	7324.8721	723.2576	709.9	38.5	7701.1238	741.0295	725.6
39.5	8082.2375	757.3654	742.2	40.5	8476.1228	774.9990	756.6
41.5	8873.1991	790.9616	771.2	42.5	9284.5880	808.4652	787.3
43.5	9697.2453	824.0462	802.2	44.5	10126.0161	841.4281	817.7
45.5	10553.8644	856.6191	832.1	46.5	10999.9039	873.8878	847.5
47.5	11442.5449	888.6805	860.5	48.5	11905.7480	905.8441	876.1
^{193}Pb(SD3)				^{193}Pb(SD4)			
10.5	639.9508			11.5	767.3806		
12.5	891.7477	251.7969	251.5	13.5	1040.7311	273.3505	273
14.5	1184.2216	292.4739	291.5	15.5	1354.1881	313.4570	313.4
16.5	1517.0037	332.7821	332.4	17.5	1707.2316	353.0435	353.1
18.5	1889.7251	372.7214	372.1	19.5	2099.3416	392.1100	391.9
20.5	2302.0170	412.2919	411.9	21.5	2529.9982	430.6566	430
22.5	2753.5105	451.4935	450.6	23.5	2998.6813	468.6831	467.1
24.5	3243.8367	490.326	488.9	25.5	3504.8710	506.1897	503.9
26.5	3772.6267	528.7900	526.6	27.5	4048.0473	543.1763	539.5
28.5	4339.5117	566.8850	563.4	29.5	4627.6902	579.6429	575.1
30.5	4944.1229	604.6112	599.9	31.5	5243.2797	615.5895	610
32.5	5586.0914	641.9685	637	33.5	5894.2958	651.0161	644.5
34.5	6265.0483	678.9569	672.2	35.5	6580.2186	685.9228	676.4
36.5	6980.6247	715.5764	709.2	37.5	7300.5281	720.3095	707.2





Rotational Frequency ħω (MeV)

Fig. 1. The calculated dynamic $J^{(2)}$ moment of inertia (closed circle) and kinematic $J^{(1)}$ (open circle) as a function of rotational frequency ħω for the three pairs of signature SD bands in ^{191}Hg , ^{193}Hg and ^{193}Pb nuclei and comparison with experimental data[3] (closed circles with error bars).

It was found that the majority of SD bands observed in odd-A nuclei in A~190 mass region are signature partner pairs of SD bands. Their bandhead moments of inertia are almost identical. To investigate the $\Delta I = 1$ staggering in signature partner pairs, the staggering function Y(I) has been calculated in terms of dipole and quadruple transition energies. The calculated values are listed in Table 3 and plotted versus the spin I in Figure (2). We notice that all signature partner pairs exhibit large amplitude staggering. The pair ^{191}Hg (SD2,SD3) is interpreted as signature partners built on the 3/2+[642] orbital, they exhibit a signature splitting of 65 keV at ħω ~ 0.4MeV.

Table 3: The calculated ΔI = 1staggering function Y(I) for the signature partner pairs ^{191}Hg(SD2, SD3), ^{193}Hg(SD3, SD4) and ^{193}Pb(SD3, SD4).

^{191}Hg(SD2, SD3)			^{193}Hg(SD3, SD4)			^{193}Pb(SD3, SD4)		
I(ħ)	Y(I) (KeV) EXP	Cal	I(ħ)	Y(I) (KeV) EXP	Cal	I(ħ)	Y(I) (KeV) EXP	Cal
12.5	0.01420	0.01466	11.5	2.9665	2.19022	12.5	-0.0528	-0.0516
13.5	-0.01802	-0.01797	12.5	-5.3712	-5.2267	13.5	0.0506	0.0496
14.5	0.016088	0.01622	13.5	5.0848	2.4776	14.5	-0.0555	-0.0526
15.5	-0.01659	-0.02031	14.5	-0.01035	-5.4862	15.5	0.0550	0.4948
16.5	0.013163	0.01918	15.5	8.3852	2.6387	16.5	-0.573	-0.0511
17.5	-0.016255	-0.02397	16.5	-0.01159	-5.6139	17.5	0.0538	0.0468
18.5	0.014040	0.02342	17.5	8.4840	2.6705	18.5	-0.0556	-0.0472
19.5	-0.01800	-0.02888	18.5	-0.01127	-5.6085	19.5	0.0508	0.0417
20.5	0.016659	0.02887	19.5	7.1125	2.5707	20.5	-0.0498	-0.0408
21.5	-0.022189	-0.03498	20.5	-0.01007	-5.4679	21.5	0.0422	0.0341
22.5	0.021490	0.03547	21.5	5.9895	2.3357	22.5	-0.0401	-0.0319
23.5	-0.028256	-0.04224	22.5	-9.0821	-5.1893	23.5	0.0306	0.0239
24.5	0.028422	0.04321	23.5	5.0514	1.9628	24.5	-0.0263	-0.0204
25.5	-0.03620	-0.05065	24.5	-8.6478	-4.7709	25.5	-0.0153	0.0111
26.5	0.036859	0.05208	25.5	3.7066	1.4494	26.5	-0.0100	-0.0063
27.5	-0.046071	-0.06020	26.5	-6.4150	-4.2102	27.5	-0.0033	-0.0043
28.5	0.047587	0.06207	27.5	-0.1631	0.7929	28.5	0.0099	0.0102
29.5	-0.05769	-0.07090	28.5	-2.6952	-3.5045	29.5	0.0238	-0.0225
30.5	0.059421	0.07318	29.5	-2.5684	-0.01012	30.5	0.0308	0.0295
31.5	-0.070336	-0.08276	30.5	-0.4179	-2.6509	31.5	-0.0461	-0.0434
32.5	0.072361	0.08543	31.5	-5.0258	-0.09621	32.5	0.0553	0.0515
33.5	-0.085017	-0.09579	32.5	1.4675	-1.6470	33.5	-0.0733	-0.0672
34.5	0.087920	0.09882	33.5	-7.6817	-2.0658	34.5	0.0821	0.0762
35.5	-0.100603	-0.10999	34.5	4.3657	-0.4901	35.5	-0.1040	-0.0939
36.5	0.103449	0.11337	35.5	-9.0839	-3.3246	36.5	0.1177	0.1037
37.5	-0.117126	-0.12540	36.5	4.5782	0.8230	37.5	-0.1480	-0.1236
38.5	0.120012	0.12909	37.5	-9.2501	-4.7416			

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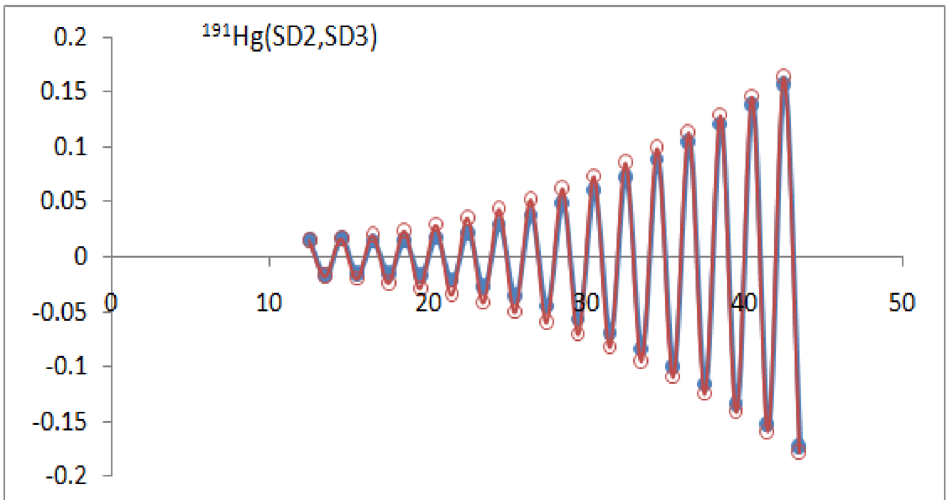
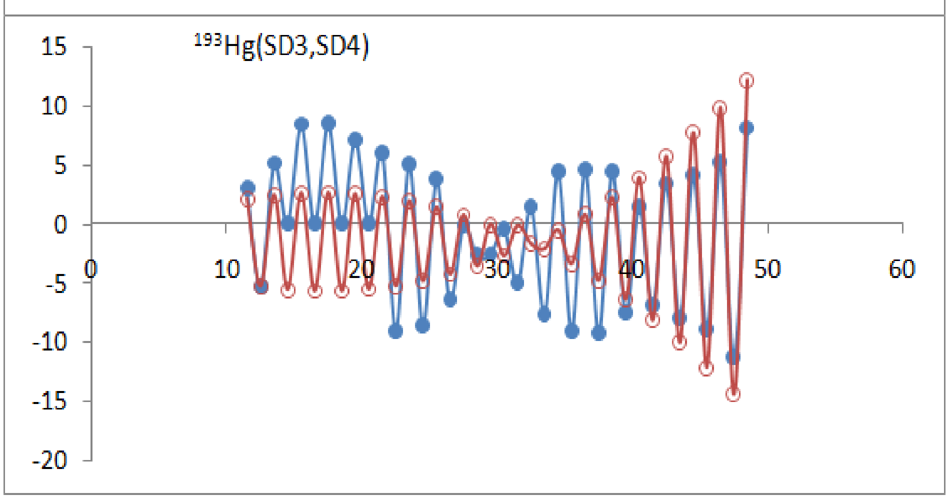
^{191}Hg(SD2, SD3)			^{193}Hg(SD3, SD4)			^{193}Pb(SD3, SD4)		
I(ħ)	**Y(I) (KeV)**		**I(ħ)**	**Y(I) (KeV)**		**I(ħ)**	**Y(I) (KeV)**	
	EXP	**Cal**		**EXP**	**Cal**		**EXP**	**Cal**
39.5	-0.13464	-0.14202	38.5	4.3730	2.2956			
40.5	0.138190	0.14599	39.5	-7.5528	-6.3198			
41.5	-0.153621	-0.15987	40.5	1.4420	3.9303			
42.5	0.156630	0.16409	41.5	-6.8755	-8.0624			
43.5	-0.174019	-0.17899	42.5	3.3760	5.7304			
			43.5	-7.9973	-9.9727			
			44.5	4.0751	7.6990			
			45.5	-8.9235	-12.0540			
			46.5	5.1945	9.8358			
			47.5	-11.2877	-14.3102			
			48.5	8.0613	12.1560			

In conclusion, the energies of three pairs signature partners in the odd SD mercury and lead nuclei are described by a simple formula including rotational, vibrational and perturbation terms. For each SD band the transition energies and level spins are parameterized by three parameter expression by using a simulated search program. The best fitted parameters have been used to calculate the rotational frequencies $\hbar\omega$, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia.





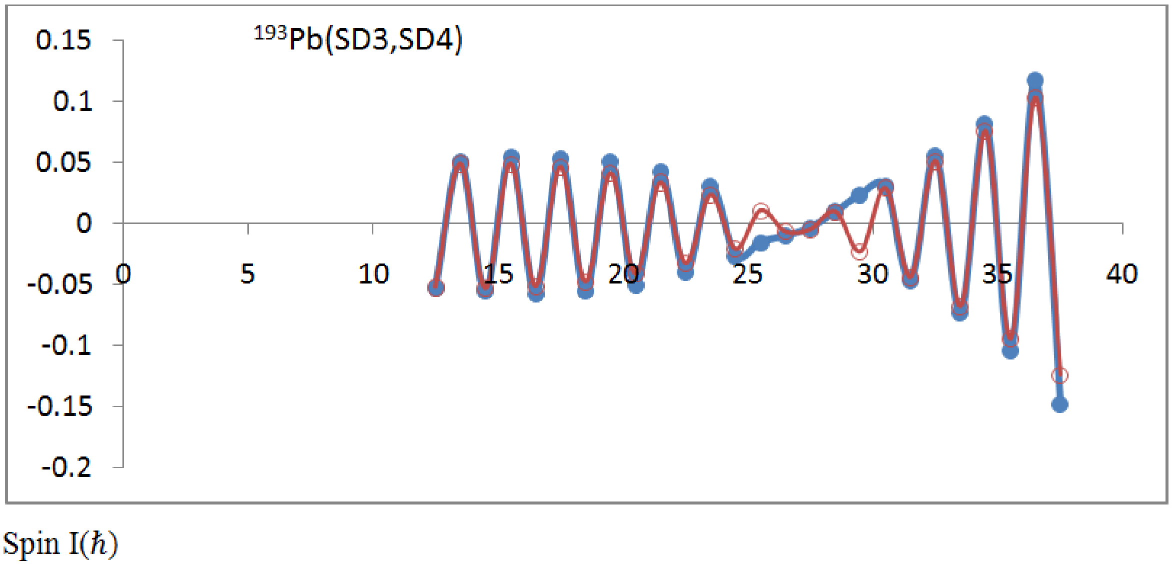


Fig. 2. The calculated staggering function Y (I) versus spin I for our selected signature partner SD bands observed in ^{191}Hg , ^{193}Hg and ^{193}Pb nuclei. The closed circles stand for signature α =+1/2 and open circles for α =-1/2.

The calculated results agree very well with the experimental data. We noticed that for all the studied SD bands $J^{(1)}$and $J^{(2)}$ are found to rise steadily with ħω and the transition energies in ^{191}Hg (SD3) is identical to ^{193}Hg (SD3). The ΔI = 1staggring phenomena in the spectrum of signature partners has been discussed by considering a staggering function includes a dipole and quadruple transition energies. A large fluctuations of the staggering function as a function of the angular momentum is found in all the considered three signature partner pairs. The staggering is very large.

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